



A Complete XVA Valuation Framework

Why the “Law of One Price” is dead

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Pricing a book of derivatives has become quite a complicated task, even when those derivatives are simple in nature. This is the effect of the new trading environment, highly dominated by credit, funding and capital costs. In this paper the author formally sets up a global valuation framework that accounts for market risk (risk neutral price), credit risk (CVA), funding risk (FVA) of self-default potential hedging (LVA), collateral (CollVA) and market hedging positions (HVA), as well as tail risk (KVA). These pricing metrics create a framework in which we can comprehensively value trading activity. An immediate consequence of this is the emergence of a potential difference between fair value accounting and internal accounting. This piece of work also explains the difference between both of them, and how to perform calculations in both worlds in a realistic and coherent manner, demonstrating via arbitrage-impossibility arguments that an XVA frameworks should be used in both cases.

During the past few years we have witnessed the birth of a number of price value adjustments. It all started with CVA, but then we moved to DVA, FVA, CollVA, KVA,

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etc [7, 3, 4, 13, 12, 10, 1, 9, 2, 6, 11]. The emergence of these pricing metrics has been somewhat irregular, naturally creating some degree of confusion in the industry.

The goal of this piece of work is to set up a formal framework for all those value adjustments, as well as to introduce some other ones generally left out in the literature. We are going to formally calculate the value of a book of derivatives, accounting for all the associated cash flows that come not only from the derivative itself, but from the act of hedging and managing default risk, funding risk and capital costs. To the author's knowledge, this is the first time that a global valuation framework, that accounts for CVA_{asset} , CVA_{liab} (DVA), CVA_{cross} , FVA , LVA , $CollVA$, HVA and KVA in a solid and comprehensive manner, is put forward.

Further to it, we are going to see how a key concept to understand derivative valuation is the difference between Price and Value. By Price it is meant the exit price that goes into a balance sheet for fair value accounting. By Value we mean how much a book of derivatives is worth to an institution. Those two concepts, being highly related, are different.

This paper demonstrates that the value of a book of derivatives is not the same for all market players, and any useful valuation framework should reflect so. We are going to see that the idea of risk-neutral valuation for fair value accounting, being a good theoretical framework, cannot be currently used because it is based in the very idea of derivative arbitrage, that cannot be exercised in the real market as that theory assumes.

We are going to start with the well known risk neutral pricing with CVA. Many readers will be familiar with it, but it is good to refresh it here to ensure the valuation framework used in subsequent sections is well understood. Further to it, we are going to extend that framework to a general XVA one, in which FVA , via LVA , $CollVA$ and HVA , together with KVA are set up. Once the XVA valuation structure is set, we are going to discuss each term to understand what they do and don't mean, as well as implications for pricing, valuation and risk management. Finally, we will compare the results to the classic risk-neutral valuation framework, and discuss why the XVA one is more appropriate at present.

The Valuation Framework

During the recent years, there has been a profusion of adjustments to the risk-neutral price of an OTC derivative, often referred to as X-Value Adjustments (XVA).

We are going to introduce the idea of XVA in two steps. First considering CVA pricing from a risk-neutral standpoint, and then introducing a derivative valuation through a full XVA framework. By 'CVA' we are going to mean the bilateral CVA.

These pricing and valuation frameworks are based in the fundamental theorem of asset pricing, that states that the fair value of a financial product "today" is the expectation

of the present value (PV) of its future cash flows.

$$P_0 = \mathbb{E} \left(\sum_i PV(\text{future cash flow}_i) \right) \quad (1)$$

We assume that there exists a ‘risk-free’ interest rate (r_u) between the time points u and $u + du$, so that the present value of a generic future cash flow (X_t) at t , to be delivered by a default-free entity, is given by

$$PV_0 = e^{-\int_0^t r_u du} X_t \quad (2)$$

In other words, $DF_{0,t} = e^{-\int_0^t r_u du}$ is the riskless discount factor.

If we are trying to value a derivative that is going to have a future cash flows $X_t = x_t dt$ between t and $t + dt$, then,

$$\sum_i PV(\text{future cash flow}_i) = \int_0^T e^{-\int_0^t r_u du} x_t dt \quad (3)$$

where T is the maturity of the derivative. Consequently,

$$P_0 = \mathbb{E} \left(\int_0^T e^{-\int_0^t r_u du} x_t dt \right) \quad (4)$$

Given that so far we live in the risk-neutral world, this price should be the same as that obtained by the Black-Scholes-Merton model. That model equates the price of a derivative (a set of future cash flows contingent on some external risk factors like, typically, interest rates, FX prices, etc) to the price of hedging out its risk and producing on this way a risk-less portfolio. In that context, the price of a derivative and its evolution is given by Equations 5 and 6.

$$\mathcal{L} \cdot P_t = r P_t - r S \frac{\partial P_t}{\partial S} \quad (5)$$

$$\mathcal{L} = \frac{\partial}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2}{\partial S^2} \quad (6)$$

Risk-neutral pricing with counterparty risk

Risk neutral pricing with counterparty risk has been well explained in the literature [7]. Let’s introduce it here to set up the subsequent valuation framework, as well as for

completeness¹.

A financial derivative is a contract between two entities, the ‘counterparties’, to exchange a number of cash flows up to the maturity date. During this time, one or both of these counterparties may default.

However, the classic ‘risk-neutral’ pricing theory does not contemplate that any market player can default. In order to incorporate this, let’s say that

- Any counterparty of the derivative can buy credit protection insurance on the other counterparty defaulting (typically in the form of a Credit Default Swap (CDS)).
- That these credit protection contracts have unlimited liquidity and no transaction costs.
- That the external entity selling them cannot default.

As it will be well known by the reader, the expectation of a generic quantity Z in the future can be obtained by summing the product of the value of Z in each possible event by the probability of each event happening. In other words,

$$\mathbb{E}(Z) = \sum_i \mathcal{P}_i Z_i \quad (7)$$

where \mathcal{P}_i is the probability of event i and Z_i is the value of Z if event i takes place.

If we have a bilateral derivative contract with a counterparty, there are four events that may happen in the future interval from t to $t + dt$, subject to both counterparties having survived up to the time point t :

1. That both counterparties are survived at $t + dt$,
2. That we survive up to $t + dt$, but our counterparty defaults during the interval $(t, t + dt)$,
3. That we default during the interval $(t, t + dt)$, but our counterparty survives up to $t + dt$,
4. That both counterparties default during the interval $(t, t + dt)$.

Let’s say that there is a ‘default intensity’ λ so that the default probability of an entity in the interval $t + dt$ is given by $\lambda_t dt$. In this framework, the survival probability of that entity up to the time point t , subject to being ‘alive’ at $t = 0$, is given by

$$S_{0,t} = e^{-\int_0^t \lambda_u du} \quad (8)$$

¹Those readers familiar with CVA may want to skip this section.

A snap shot of the four possible events we are facing, with their probabilities (\mathcal{P}_i)² and the cash flows that would occur in each of them, is shown in the following table

Event	\mathcal{P}_i in $(t, t + dt)$	Cash flow
1	$S_{t,t+dt}^{our} S_{t,t+dt}^{cpty}$	$x_t \cdot dt$
2	$S_{t,t+dt}^{our} \lambda_t^{cpty} dt$	$-(1 - RR_t^{cpty}) P_t^+$
3	$\lambda_t^{our} dt S_{t,t+dt}^{cpty}$	$-(1 - RR_t^{our}) P_t^-$
4	$\lambda_t^{our} \lambda_t^{cpty} dt$	$-(1 - RR_t^{cpty}) P_t^+ - (1 - RR_t^{our}) P_t^-$

where $x_t dt$ is the cash flow that takes place in the derivative in the interval $(t, t + dt)$ if no default happens, P_t is the price of the derivative at time t ³, $P_t^+ = \max(P_t, 0)$, $P_t^- = \min(P_t, 0)$ and RR represents the recovery rate obtained by the surviving party when a default occurs⁴.

If we say that the survival probability in an infinitesimal time step $S_{t,t+dt} \simeq 1$, and noting also that the probability of all these events must be multiplied by the probability of both counterparties having survived at t

$$S_{0,t}^{our} S_{0,t}^{cpty} = e^{-\int_0^t (\lambda_u^{our} + \lambda_u^{cpty}) du} \quad (9)$$

then the price of the derivative that accounts for counterparty risk is given by

$$P_0^{CptyRisk} = \mathbb{E} \left(\int_0^T e^{-\int_0^t (r_u + \lambda_u^{our} + \lambda_u^{cpty}) du} x_t dt \right) - \quad (10)$$

$$\mathbb{E} \left(- \int_0^T e^{-\int_0^t (r_u + \lambda_u^{our} + \lambda_u^{cpty}) du} \lambda_t^{cpty} (1 - RR_t^{cpty}) P_t^+ dt \right) - \quad (11)$$

$$\mathbb{E} \left(- \int_0^T e^{-\int_0^t (r_u + \lambda_u^{our} + \lambda_u^{cpty}) du} \lambda_t^{our} (1 - RR_t^{our}) P_t^- dt \right) - \quad (12)$$

$$\mathbb{E} \left(- \int_0^T e^{-\int_0^t (r_u + \lambda_u^{our} + \lambda_u^{cpty}) du} \lambda_t^{our} \lambda_t^{cpty} \left((1 - RR_t^{cpty}) P_t^+ - (1 - RR_t^{our}) P_t^- \right) dt \right) \quad (13)$$

²Remembering that these \mathcal{P}_i are subject to both counterparties having survived up to time t .

³Strictly speaking, P_t should be the replacement value of an equivalent derivative should a default occur.

That replacement trade would be with a counterparty with equivalent credit quality of the defaulted entity. This leads to two problems: firstly, what 'credit quality' should we use? one second before the company defaults? one year before? This is not clear. Secondly, this creates a mathematical recursive loop, as P_t should contain also a counterparty risk adjustment. It is market practice to ignore this refinement in the calculation because it is very difficult to solve and, importantly, it hardly makes any relevant difference in most practical cases.

⁴It should be noted that both cash flows in terms 2 and 3 must have a negative sign. In the case of 2, because it is a loss that we could incur. In the case of 3, it is a net gain, but P_t^- is a negative number, and so it needs a negative sign to counteract it.

where we have sum across all possible time points in t^5 . Each of those terms can be called P_0^* (Eq. 10), CVA_{asset} (also known as CVA, Eq. 11), CVA_{liab} (also known as DVA, Eq. 12) and CVA_{cross} (Eq. 13).

Often it is assumed that event 4 in the table above has a negligible probability; i.e., that both counterparties joint default probability is nearly zero ($\lambda_t^{our} \lambda_t^{cpty} dt \simeq 0$) and, consequently, $CVA_{cross} = 0$. If the correlation between our default event and our counterparty's is relevant (e.g., two similar institutions in the same country), we may have to keep that term. In the extreme case in which that correlation is close to one⁶, then events 2 and 3 can become negligible, and it is event 4 the one that is most important.

Let's focus in the standard case, negligible event 4. If we say now that $DF_{0,t}^* = e^{-\int_0^t (r_u + \lambda_u^{our} + \lambda_u^{cpty}) du}$ is the risky discount factor and that the recovery rates are constant over time, then

$$P_0^{CptyRisk} = \mathbb{E} \left(\int_0^T DF_{0,t}^* x_t dt \right) - \quad (14)$$

$$- \mathbb{E} \left((1 - RR^{cpty}) \int_0^T DF_{0,t}^* \lambda_t^{cpty} P_t^+ dt \right) - \quad (15)$$

$$- \mathbb{E} \left((1 - RR^{our}) \int_0^T DF_{0,t}^* \lambda_t^{our} P_t^- dt \right) \quad (16)$$

Furthermore, let's assume now that the discount factors are independent of x_t and of P_t and that default events are independent of x_t and P_t (i.e., that there is no right or wrong way risk). Then,

$$P_0^{CptyRisk} = \int_0^T DF_{0,t}^* \mathbb{E}(x_t) dt - \quad (17)$$

$$- (1 - RR^{cpty}) \int_0^T DF_{0,t}^* \lambda_t^{cpty} \mathbb{E}(P_t^+) dt - \quad (18)$$

$$- (1 - RR^{our}) \int_0^T DF_{0,t}^* \lambda_t^{our} \mathbb{E}(P_t^-) dt \quad (19)$$

If now we define the Expected Positive Exposure as $EPE_t = \mathbb{E}(P_t^+)$ and the Expected Negative Exposure as $ENE_t = \mathbb{E}(P_t^-)$, then

⁵I.e., integrated in continuous time.

⁶E.g., our counterparty is a sister company of ours, in another country, but being a different legal entity.

$$P_0^{CptyRisk} = \int_0^T DF_{0,t}^* \mathbb{E}(x_t) dt - \quad (20)$$

$$-(1 - RR^{cpty}) \int_0^T DF_{0,t}^* \lambda_t^{cpty} EPE_t dt - \quad (21)$$

$$-(1 - RR^{our}) \int_0^T DF_{0,t}^* \lambda_t^{our} ENE_t dt \quad (22)$$

The first term (20) is the ‘classic’ risk neutral price under the risky discounting measure, the second term (21) is the asset side of CVA (a.k.a. CVA), and the third term (22) is the liability side of CVA, (a.k.a. DVA).

$$CVA_{asset,0} = (1 - RR^{cpty}) \int_0^T DF_{0,t}^* \lambda_t^{cpty} EPE_t dt \quad (23)$$

$$CVA_{liab,0} = (1 - RR^{our}) \int_0^T DF_{0,t}^* \lambda_t^{our} ENE_t dt \quad (24)$$

Therefore,

$$P_0^{CptyRisk} = P_0^* - CVA_0 \quad (25)$$

$$CVA_0 = CVA_{asset,0} + CVA_{liab,0} \quad (26)$$

If s_t is the credit spread of the CDS of a given entity, it is quite common to say that $s_t \simeq (1 - RR) \lambda_t$. Then, in this context,

$$CVA_{asset,0} \simeq \int_0^T DF_{0,t}^* s_t^{cpty} EPE_t dt \quad (27)$$

$$CVA_{liab,0} \simeq \int_0^T DF_{0,t}^* s_t^{our} ENE_t dt \quad (28)$$

And, finally, if s_t is a fairly constant number, and we define

$$\widehat{EPE}_0^* = \int_0^T DF_{0,t}^* EPE_t dt \quad (29)$$

$$\widehat{ENE}_0^* = \int_0^T DF_{0,t}^* ENE_t dt \quad (30)$$

then,

$$CVA_{asset,0} \simeq \widehat{EPE}_0^* \cdot s^{cpty} \quad (31)$$

$$CVA_{liab,0} \simeq \widehat{ENE}_0^* \cdot s^{our} \quad (32)$$

Sometimes it is also common practice to neglect the ‘riskiness’ of the discount factors. This is a good approximation when both counterparties are entities with good credit standing, and when the book of trades between them doesn’t mature too far in the future. In these cases,

$$CVA_{asset,0} \simeq s^{cpty} \cdot \int_0^T DF_{0,t} EPE_t dt \quad (33)$$

$$CVA_{liab,0} \simeq s^{our} \cdot \int_0^T DF_{0,t} ENE_t dt \quad (34)$$

or

$$CVA_{asset,0} \simeq \widehat{EPE}_0 \cdot s^{cpty} \quad (35)$$

$$CVA_{liab,0} \simeq \widehat{ENE}_0 \cdot s^{our} \quad (36)$$

Derivative valuation with counterparty, funding and capital risk

A central concept that we are going to introduce now is the difference between *Price* and *Value*.

The pricing framework just seen provides a *risk-neutral* price with counterparty risk. The *Price* of a derivative tries to capture for how much should two generic institutions trade the derivative in a ‘fair’ way, with no consideration of the specific environment they operate in. In contrast, the *Value* of a derivative tries to capture that specific environment.

Let’s define the *Value to Me* (*VtM*) as

$$VtM = P_{sale} - P_{manufacturing} \quad (37)$$

where P_{sale} is the expectation of the present value of the future cash flows in the deal with the counterparty, and $P_{manufacturing}$ is the expectation of the present value of the future cash flows in the activities we need to ‘manufacture’ or manage the risk of the trade. These activities are going to be managing the effects of market, counterparty, funding and tail risk. This will be done by hedging out the risk intrinsic in the trading markets when possible, or by perhaps accruing a risk reserve for it when it is non-hedgeable.

The Selling Price

If we are a dealer and we want to sell a derivative to a client, we start our valuation following the Black-Scholes-Merton thinking process, saying that there are a collection of financial positions that we can set against the markets, and that carry the same but symmetrical market risk than the original one that we want to sell to the client. Those trades are the so-called *hedging* positions.

Let's simplify the language and assume for now that one single 'back-to-back' position hedges the derivative perfectly. That hedging position is going to have a cash flow $x_t dt$ in the time interval $(t, t + dt)$.

Typically, a derivatives dealer is going to put a spread on top of that hedging cash flow, where it makes its profit from. On this way, the cash flows in the derivative sold to the client are given by $(x_t + \delta_t) dt$. Therefore,

$$P_{sale} = \mathbb{E} \left(\int_0^T DF_{0,t}^* (x_t + \delta_t) dt \right) \quad (38)$$

The Manufacturing Price

On the manufacturing side of the equation, we have four components:

$$P_{manufacturing} = P_{MarketRisk} + P_{CounterpartyRisk} + P_{FundingRisk} + P_{CapitalRisk} \quad (39)$$

Let's get into each of those terms.

- **Market Risk**

We have said that the cash flows in the hedging position is going to be $x_t dt$ in the time interval $(t, t + dt)$. Hence,

$$P_{MarketRisk} = \mathbb{E} \left(\int_0^T DF_{0,t}^* x_t dt \right) \quad (40)$$

Strictly speaking, the discount factors in Equations 38 and 40 should be different, as the counterparties in the bilateral and hedging positions are not the same. However, that refinement results most often in a second order adjustment that we are going to ignore for now.

- **Counterparty Risk**

We have seen that CVA represents the cost of hedging out counterparty risk. The asset side of CVA can be hedged by buying a series of CDS in the market, so that

$$CVA_{asset} \simeq \widehat{EPE}_0^* \cdot s^{CDS,cpty} \quad (41)$$

However, the liability side of CVA is a different story, as it represents the cost incurred to hedge our own default. The way our counterparty can do that is by buying a CDS on us, hence paying our credit spread ($s^{CDS,our}$) for that credit insurance.

However, critically, that cost is totally irrelevant to us, as that is not a cost that we have. The cost that we have to hedge out our own default would be *borrowing* today the cash we expect to need to pay in the future, and putting it aside so that we can use it as time progresses. The cost of doing that self-hedge is given by our funding spread, which is the credit spread (s^{our}) plus a liquidity spread (l^{our})⁷. As a result, *our* cost of hedging *our own* default is given by

$$CVA_{liab} \simeq \widehat{ENE}_0^* \cdot (s^{CDS,our} + l^{our}) \quad (42)$$

as the *ENE* reflects our expected liabilities.

Putting all this together,

$$P_{CounterpartyRisk} = \widehat{EPE}_0^* \cdot s^{CDS,cpty} + \quad (43)$$

$$\widehat{ENE}_0^* \cdot s^{CDS,our} + \quad (44)$$

$$\widehat{ENE}_0^* \cdot l^{our} \quad (45)$$

Equation 43 leads to the asset side of CVA, Equation 44 to the liability side of CVA, and Equation 45 to a Liquidity Value Adjustment (LVA).

$$CVA_{asset} = \widehat{EPE}_0^* \cdot s^{CDS,cpty} \quad (46)$$

$$CVA_{liab} = \widehat{ENE}_0^* \cdot s^{CDS,our} \quad (47)$$

$$LVA = \widehat{ENE}_0^* \cdot l^{our} \quad (48)$$

⁷Strictly speaking, that cost is given by the full funding rate ($r^{RiskFree} + s^{our} + l^{our}$), but we can remove the risk-free rate from it as, in principle, we can deposit those funds into a quasi-non-defaultable entity, that should return the risk-free rate back to us. So the net cost is the funding spread ($s^{our} + l^{our}$).

LVA This last LVA terms could be seen as a funding risk term, as it is a cost that is attached to the liquidity-funding premium. This number reflects the fact that the bond market has a different liquidity environment to that of the CDS market. Basically, two institutions can agree on a credit insurance contract just by signing the respective CDS document, but if one wants to buy or sell an actual bond with that same credit risk, that bond needs to be found somewhere, and there is an actual limited availability of them. In other words, they are not infinite in the real life, as the Black-Scholes-Merton model assumes. Hence LVA is an adjustment that we must do to the risk-neutral value of a portfolio of trades to account for the real liquidity constraints that we face in the funding and credit market, and it reflects the difference between our counterparty's cost of hedging our risk Vs. our own cost of hedging that same risk.

- **Funding Risk**

In addition to the liquidity-funding risk that we have just seen, there are two other sources of funding risk to be considered.

CollVA An important credit risk mitigant is collateral, that is posted and received constantly between financial institutions. Every day, we are going to have to fund the net collateral that we have to post (or lend out the collateral we receive) from *all* our trading positions. This must include the OTC derivatives, hedging positions, positions in clearing houses, etc. If s_t^{borrow} and s_t^{lend} are the spread over the risk-free rate at which we can borrow and lend unsecured cash, and if s^{post} is the spread over the risk-free rate that we are charged on collateral posted, then, following the same idea of the CVA derivation, the Collateral Cost Adjustment (*CollCA*) and Collateral Benefit Adjustment (*CollBA*) are

$$CollCA_0 = \int_0^T EPE_t^{collateral} \cdot DF_t^* \cdot (s_t^{borrow} + s^{post}) dt \quad (49)$$

$$CollBA_0 = \int_0^T ENE_t^{collateral} \cdot DF_t^* \cdot s_t^{lend} dt \quad (50)$$

where $EPE_t^{collateral}$ and $ENE_t^{collateral}$ represent the expected positive and negative exposure of the net collateral needs.

The collateral requirements are often split into an Initial Margin and a Variation Margin. They are also going to depend very strongly on CSA agreements, re-hypothecation conditions, exchange and clearing houses margining requirements, etc.

In this context,

$$CollVA = CollCA + CollBA \quad (51)$$

In order to calculate this *CollVA*, in practice, we need to simulate the collateral in a Monte Carlo simulation, with all its peculiarities like rehypotication, demands from central and standard counterparties, etc, net it at the appropriate funding-set level, which is often the whole portfolio of derivatives and goes beyond netting sets, to come up with a net collateral needs per scenario and time point in the future, from which we can calculate the collateral profiles $EPE_t^{collateral}$ and $ENE_t^{collateral}$, and subsequently compute Equations 49 and 50.

HVA In reality, often we do not have perfect ‘back-to-back’ hedges. Hence we are going to have an additional funding adjustment from the difference in cash needs from the trade that we have with a counterparty and the position that we buy in the market to hedge its market risk. Important to note, this is *regardless* of any collateral arrangements. For example, we may hedge a 10-year swap with annual coupons sold to a client with a 10-year swap with quarterly coupons. If $(x_t + h_t) dt$ represents the cash flows in the actual hedging trade, where x_t would be the cash flows from the perfect hedging trade, then, we need to fund the extra h_t if we need to borrow it, or we can lend it out too if we have excess of it. This means that we are going to have two new adjustments: Hedging Cost Adjustment (*HCA*) and Hedging Benefit Adjustment (*HBA*) that, following again the ideas previously expressed, are

$$HCA_0 = \int_0^T EPE_t^h \cdot DF_t^* \cdot s_t^{borrow} dt \quad (52)$$

$$HBA_0 = \int_0^T ENE_t^h \cdot DF_t^* \cdot s_t^{lend} dt \quad (53)$$

where EPE_t^h and ENE_t^h represent the expected positive and negative exposure of the extra hedging cash needs. Then,

$$HVA = HCA + HBA \quad (54)$$

FVA Given that we have seen that there are three sources of funding risk: funding-liquidity (*LVA*), Collateral funding (*CollVA*) and Hedging funding (*HVA*), we can put all this together into one term to simplify things somewhat.

$$FVA = CollVA + LVA + HVA \quad (55)$$

Most often, *CollVA* is seen as the most important source of funding risk, but this may not always be the case.

- **Tail Risk Capital** represents a real cost for an organisation and it measures the tail risk it faces. This cost is clearly tangible in regulated financial institutions as they need to do a capital allocation in their balance sheet as dictated by their regulators, but it could also be the case in un-regulated organisations if they want to build a risk reserve against unexpected losses. In this context, following again the thinking process shown before, we can see KVA as

$$KVA_0 = \int_0^T EK_t \cdot DF_t^* \cdot r_{c,t} \cdot dt \quad (56)$$

where EK_t is the expected capital at time t and r_c is the rate of cost of capital that the institution has. A good candidate for this r_c could be the Weighted Average Cost of Capital (WACC).

Putting all this together, the manufacturing cost of a book of derivatives is going to be given by

$$P_{manufacturing} = \mathbb{E} \left(\int_0^T DF_{0,t}^* x_t dt \right) + CVA + FVA + KVA. \quad (57)$$

The Value to Me

Blending together Equations 37, 38 and 57, the *Value to Me* of a derivative is given by

$$VtM = Profit_{RiskNeutral} - CVA - FVA - KVA \quad (58)$$

where

$$Profit_{RiskNeutral} = \int_0^T DF_{0,t}^* \mathbb{E}(\delta_t) dt \quad (59)$$

This value is most important for trading decision-making, incentives setting and senior management. Value tells a trading unit what is the minimum price at which it needs to deal a derivative to be economical. Then, it will obviously try to trade at the highest possible price possible, in order to maximise share holders value.

The Break-even Point

For a trade to be economical for a dealer, the *VtM* must be positive. As a result, the spread δ_t needs to be so that

$$Profit_{RiskNeutral} > XVA \quad (60)$$

where $XVA = CVA + FVA + KVA$.

From a derivatives user stand point, VtM is typically going to be a negative number. It needs to be small enough so that this VtM cost is worthwhile related to the real risks it hedges.

Relation to the Black-Scholes-Merton Risk-neutral World

In the classic Black-Scholes-Merton risk-neutral theoretical world,

- $XVA = 0$, as there are no default, funding or capital risk.
- $\delta_t = 0$, as we are looking for the 'fair' price.
- $Profit_{RiskNeutral} = 0$, due to the Law of One Price.

Consequently, $VtM = 0$ in that framework.

XVA in everyday activities

Before a trade is incepted, P_{sale} in Equation 37 is a floating variable. If we are a dealer, we need to calculate $P_{manufacturing}$ and make sure that the spread δ that we apply to the trade is big enough so that VtM is sufficiently large.

Once the trade is incepted, δ is fixed, and then we should calculate VtM periodically to make sure it the book of trades is economical. If VtM goes to negative territory, it may be sensible to unwind the book as it is expected to make a loss going forward.

Discussion

This framework yields a number of interesting points for discussion.

The meaning of each XVA term

Let's go through each of the XVA terms we have seen, to understand what they do and don't mean. As the reader may anticipate, a most tricky part is going to come from FVA and its potential overlap with CVA_{liab} .

- **The Asset side of Default Risk**

CVA_{asset} represents our expected cost from hedging the default risk we are exposed to. We may or may no hedge that risk. If we hedge it, CVA_{asset} represents the

present value of the expected costs. If we do not hedge it, it represents the gain we will make *if* the counterparty does not default because we are assuming that default risk.

- **The Liability side of Default Risk**

This term represent the expected cost from hedging our own default risk. From the counterparty standpoint this cost is $CV A_{liab}$, or DVA . However, from our point of view, this term is $CV A_{liab} + LVA$, as on that way we account for the liquidity premium we have to pay if we want to hedge our own default, in a given netting set, by forward-borrowing its expected liabilities. This is because we do not have access to our own CDS, but we have access to our own borrowing market.

It is most important to note that this term, really, is never a true cost to a financial institution, as all of them have geared balance sheets; nobody borrows forward all its expected future liabilities. In fact, banks make money from managing and running their own default risk by other means⁸.

Consequently, $CV A_{liab} + LVA$ is not a true cost we see, but an opportunity cost. This is central.

- **Funding Cost of Collateral ($CollVA$)**

This term is, indeed, a very important source of cost these days. It reflects the actual cost of borrowing the collateral we need to post as well as, potentially, the benefit of lending out the excess collateral we may have. This number must be calculated at Funding Set level, which is a business unit that is intended to be self-sufficient from a funding point if view. In most financial institutions, the whole book of derivatives tends to be considered a Funding Set.

Importantly, this number is highly driven by re-hypothecation agreements, as collateral received that cannot be re-hypothecated may decrease default risk, but does nothing to funding risk: it is like un-collateralised from a funding stand point.

- **Funding Cost of Hedging (HVA)**

In many cases, derivatives are hedged back-to-back: given a trade with a counterparty, we have an *identical* but opposite trade somewhere else so that we are overall market neutral. However, reality is not always as simple as that.

Some times we hedge a 10-year swap with annual payments with a 10-year swap with monthly coupons. Consequently we are going to have a miss-match in the cash flow requirements, that could create an additional funding cost or benefit. HVA accounts for that.

This term tends to be neglected by many financial institutions because it is often

⁸Via market, default, funding and tail risk hedging.

small⁹. However, that is not always the case. For example, let's think of a power derivative. We agree with a client to deliver power¹⁰, Sunday to Thursday, from 1 am to 6 am. We cannot hedge that perfectly in the market. A trading desk will try to match the value of that contract with standard Base and Peak power contracts, that deliver power for either twenty four or twelve hours (8 am to 8 pm, in weekdays) per day. Obviously the cash flows¹¹ in the derivatives and its hedging positions are going to be very different even when their respective P&L may be quite symmetric. That difference needs to be funded, and that funding cost is accounted for in *HVA*.

In the banking industry, this *HVA* could also be important for highly exotic book of trades, as the delta-hedging position try to match the P&L fluctuations, but cash flows could be quite misaligned.

To be noted, this Hedging Value Adjustment happens regardless of any collateral arrangements. However, strictly speaking, its related cash requirements could be further netted with collateral needs.

- **Cost of Capital (*KVA*)**

All financial institutions know that capital creates a true tangible cost that we currently need to face, the cost being the return that investors expect¹². Trades that create a capital release should be encouraged at the expense of those that create capital consumption.

Also, a trading house, that perhaps does not operate under the tight regulatory capital conditions banks currently do, should consider this term too. Let's think of two trades, with same value in expectation (i.e., same added risk neutral, credit and funding valuations), but one of them creates more tail risk than the other one. Should they have the same value for an organisation? Obviously, the answer is "No".

Any financial institution should have a capital model to manage the tail risks it faces. For example, typically, the derivatives trading unit in a corporate has a credit line from its treasury with a given limit. Let's say that that limit is of \$100m. Given that that unit cannot hedge forward its future funding costs, as its treasury only offers only a short-term borrow and lending facility, it makes sense that it discourages those trades that increase the probability of future funding needs going beyond \$100m, and encourages those that decrease it. In other words, it wants to decrease tail funding risk. The appropriate capital model and *KVA* charge will do this automatically and naturally.

Another equivalent way of looking at this is seeing that tail risk is created by the

⁹And very difficult to calculate.

¹⁰Either physically or cash-equivalent.

¹¹Or power flows.

¹²Via equity, debt, etc.

risks that we are not hedged against, and KVA is the price that we put to it.

Sources of FVA

The term FVA has now become a standard in the industry. The main source of funding cost that banks have now is that coming from its collateral needs. This is captured by $CollVA$. For this reason, quite often the term FVA has become equivalent to $CollVA$.

This is OK in most cases, because often the other two funding-related terms (LVA and HVA) are either neglected because it is a non-real opportunity cost (LVA) or negligible because it is small (HVA). However, they must not be completely forgotten as, in some instances, it may make sense to account for them.

Funding Double Counting

There has been a lot of debate about how to avoid funding double counting. That debate is based in the confusion created around the fact that $CV A_{liab}$ seems to account for our funding, but FVA (i.e., $CollVA$ in most cases) too.

Let's clarify this with the following points.

- $CV A_{liab}$ is our *counterparty's* cost of hedging our own default. That is a cost that we do not incur in, so we can forget about it in our valuation.
- $CV A_{liab} + LVA$ is our *own* cost of hedging our own default via borrowing the expected future liabilities for each nettings set, and putting them "aside" for our cash outgoings. No bank does that, because all banks are currently highly geared institutions. Hence, this is not a real cost but an opportunity cost.
- $CollVA + HVA$ is our cost of managing our own default via hedging. We post (and receive) collateral to give (and receive) guarantees that if someone defaults, it will be as close as 'business as usual' as possible. This is a real cost.

For this reason, accounting for both ($CV A_{liab} + LVA$) and $CollVA + HVA$ is often seen as double-counting the funding cost. In this context, ($CV A_{liab} + LVA$) should be dropped.

Relative Size of XVAs

We have condensed all XVA terms into three: CVA that accounts for pure credit risk, FVA that accounts for pure funding risk, and KVA that considers the cost of capital or tail risk. A central question is, should we use all of them in our valuation?

The answer to that is that it all depends in their relative size. If CVA is, say, 100 bps, FVA 50 bps, and KVA 0.1 bps, then we should neglect KVA. However, all of them should be considered in principle, and only disregarded when it is clear that its size is negligible. Green *et al.* show that a simple 10-year swap can have relevant CVA, FVA and KVA [6].

Incremental Marginal Costs and Benefits

One of the main reasons why an XVA framework is so useful is because it naturally delivers the right incentives in an organisation. It is best practice in business management that the incremental marginal costs that an activity creates must be accounted for in its cost & benefit analysis.

An XVA framework offers an ideal scenario for that in the context of derivative trading. When a decision is being made regarding a new trade coming in, or the possibility of a trade unwind, the incremental XVA is going to provide the marginal real and tangible cost that will be subsequent to the decision. On this way trading activity truly reflects the economics of the decision in the global manner that is needed.

Consequently, when we talk about XVA we can refer to two things: the overall XVA, that is relevant to the whole book of derivative and, hence, should be relevant for accounting, and the incremental XVA, that is relevant to the trading unit, in which trading activity should be based on.

Trade Valuation within a Portfolio

One of the consequences of this valuation framework is that the value of a trade depends of the context it is traded in. This is qualitatively illustrated in Figure 1.

Risk	Charge	Unsecured	Secured, no rehypothication	Secured, with rehypothication	CCP
Credit	CVA	H	M	M	L
Funding	FVA	H	H	L	M
Capital	KVA	H	M	M	L
Total	XVA	⇒ This number dictates new trading incentives			

● H High cost ● M Medium cost ● L Low cost

Figure 1: Qualitative illustration of the XVA charges, subject to different trading conditions. Either totally unsecured (no collateralisation), secured via collateral agreements in which rehypothication may or may not be allowed, and over-secured via trade novation to a Central Counterparty (CCP).

This is one of the key strengths of this valuation framework: it reflects market reality. At present, in the real world, buyer and sellers are willing to trade the same derivative at different price depending on the context they are traded in¹³. Hence, the valuation of a single trade is not a task related to that trade in isolation any more, but it is now a full portfolio value calculation.

Risk Management

This has fundamental implications for risk management in financial institutions. In the past, risk was mostly managed on a limits basis: limiting VaR, limiting potential exposure, limiting stress metrics, etc. That has led to an often difficult relationship between the trading units and the risk departments, as their respective standpoints were quite different.

With this XVA framework, things change profoundly. Now the task is to set a price (XVA) to the risk, that reflects the true risk taken by a book of derivatives. Then, it is up to the trading unit to go with it or not. In this world, risk managers do not need to go around ‘policing’ trades, saying ‘yes’ or ‘no’ to them; rather, they put a price to the risk of a trade, and the trading desk decides¹⁴.

Consequently, XVA managers are experts in calculating price-to-risk and, so, that is the center of their activity, leaving actual trading decisions to those that are experts in that: traders.

Price Vs. Value

Let’s understand the difference between Price and Value.

Value (i.e., *Value to Me*) tries to assess whether a derivative, or a book of them, is economical or not, hence it should be used to make trading decisions and for internal incentives in an organisation. However, by Price we mean the price of a book of derivatives in a balance sheet. This is an accounting metric and, so, being related to the Value, it is not the same. Price is based in fair-value accounting, which is subsequent based in the “exit” price price of a book of derivatives.

The exit price of a single derivative or a book of them is a theoretical number, that is only rarely realised. The difficult task that accountants have is trying to calculate that theoretical exit price. That task becomes even more difficult when we realise that, most often, exit prices are often exercised under special market conditions and in ‘bulk’¹⁵, as

¹³It is most important to realise that this is not a theoretical assumption or result, it is a market fact.

¹⁴Obviously, limits may also co-exist, but they can be managed now differently. For example, trades that are close or even above certain limits could have a premium XVA charge to compensate for the extra risk they generate.

¹⁵Many trades together.

opposed to trade-by-trade and in an orderly manner as the construction of fair value usually accounting does.

And this is where the debate starts. Some think that fair value accounting should account for FVA nor KVA. Some think it should not [8]. Basically, the argument of those in favour is that when a market player is thinking of buying a book of trades from another player, it is going to put into its valuation its own funding and capital costs, so fair value accounting should account for it. The argument of those against it is that, by doing so, the same derivatives will have different prices for different players; this will lead to arbitrage opportunities that could be exploited. When so, the price will be driven to that “fair” market value that we are looking for.

In the author’s view, the argument of those against FVA and KVA in the exit price may be theoretically sound, but it fails to reflect reality because current markets do not allow for that theoretical arbitrage to be exercised.

Arbitrage Opportunities

There is substantial anecdotal evidence that arbitrage cannot be exercised as the Black-Scholes-Merton model assumes. Risk Magazine published an article in September 2014 that provides strong evidence that traditional arbitrageurs are exiting the market due to increasing costs [5], mostly liquidity constraints driven by funding and capital costs. In another article titled ‘BoE’s Conliffe: era of free liquidity is over’, reportedly Conliffe from the Bank of England has said, regarding market liquidity, that “It was there, freely available and you could sell what you wanted, when you wanted, where you wanted, until one day you couldn’t and the whole system came to a crash. . . We are not going back to that. People will have to pay more”. Finally, the author of this paper started his career in finance in a hedge fund, precisely exploiting miss-prices in complex derivative products; the strategies that he used to implement, and that were very successful at the time, would be impossible to implement today due to the increased trading costs.

If there existed a true arbitrage market, then that risk-neutral fair price would be there, and hence the goal of accounting would be to calculate it. However, the problem we face is that that price does not exist, because the bracket of prices inside of which arbitrage cannot be exercised has become quite large, and hence there is a wide range of prices that are ‘correct’. Consequently, if we cannot distinguish between the correctness of two prices, the natural conclusion is that both are correct. We may like more or less that idea, perhaps feel unease by it, but that is how the market is today.

Arguably, those arbitrage-brackets have always existed, as there has always been trading friction in the market. The difference is that those brackets used to be fairly narrow, hence giving the impression that a risk-neutral price existed. However, currently, those brackets are so wide that arbitrageurs cannot exploit theoretical opportunities. Furthermore, everything suggests that those brackets are to remain for the foreseeable future, as very clearly pointed out by Conliffe.

Fair Value Accounting

This has left accountants with the difficult task of calculating a number, the risk-neutral fair value, that does not exist in the real world.

The author is not an accountant, so he prefers to leave the details of that difficult problem to expert in that field, but it seems sensible to say that what we should aim is to come up with a fair exit-price valuation framework that accounts for how a potential buyer of a book of derivatives will value it. That potential buyer will naturally put into the equation its own funding and capital costs, hence we should somehow put them too when calculating an exit price.

The problem is that those funding and capital costs are not unique, they are very institution dependent. Hence all we can reasonably aim for is to calculate an *estimation* of them, using blended averages approaches like an ‘average market funding rate’. That seems to be the framework that is being adopted by a number of market players [2]¹⁶.

The ‘Law of One Price’ Does Not Hold Any More

A consequence of all this is that one of the pillars of derivative pricing in the past, the ‘law of one price’, does not hold any more. Hence, a new pricing and valuation framework needs to be implemented. The author hopes this paper helps in that regard.

A further consequence is that, if we still try to price derivatives with the classical Black-Scholes-Merton model, each institution needs to use its own risk-neutral measure that reflects its funding and capital cost. In other words, there isn’t a market-wide risk-neutral measure for derivatives pricing. This has been explained in detail by Kenyon and Green [11].

Conclusions

We have seen a complete XVA valuation framework that accounts for the market risk, credit risk, funding risk and tail risk of a book of derivatives. We have seen that this framework should be used to calculate the *Value to Me* of a book of trades, that should be the driver of trading decisions and internal incentives.

Credit risk tends to be decomposed into its asset and liability side. However the liability side, together with the credit liquidity premium, could be dropped because it does not reflect a true tangible cost; it is an opportunity cost.

¹⁶One of the problems of the current market practices is that there isn’t a market consensus in this area yet, so balance sheet comparison are difficult if not impossible. One of the tasks of accounting bodies going forward should be to create a levelled playing field for all market participants in this space.

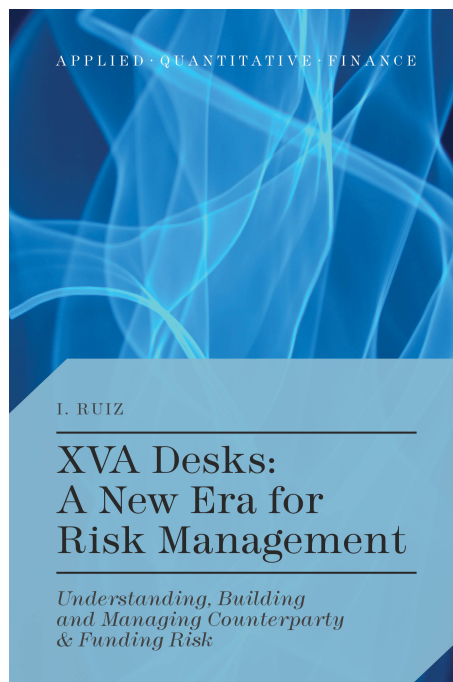
Funding risk tends to be dominated by the cost and benefit of collateral, but an extra term to account for imperfect hedging strategies may make sense in some cases.

Capital costs reflect the price of managing the tail risk that institutions face. Currently, that cost is driven by the liquidity and capital regulatory requirements, but an institution without those constraints should also implement a capital model to reflect its intrinsic tail risk.

We have also seen that fair-value accounting for balance sheet should also be based in this XVA valuation framework, as the classical risk-neutral Black-Scholes-Merton model fails to describe market reality due to the high cost of exercising derivative arbitrage strategies. The difference between internal and fair-value accounting is that, for the former, we have to use our own specificities for XVA calculation, while for the later we may need to estimate a ‘blended’ market XVA as the best estimate we can achieve for a somewhat realistic exit price.

XVA Desks - A New Era for Risk Management

All these ideas, in much more detail, together with a comprehensive look at XVA calculations, can be found in the book *XVA Desks - a New Era for Risk Management* by Ignacio Ruiz.



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