Modelling Credit Spreads for Counterparty Risk: 
Mean-Reversion is not Needed*

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Abstract

When modelling credit spreads, there is some controversy in the market as to whether they are mean-reverting or not. This is particularly important in the context of counterparty risk, at least for risk management and capital calculations, as those models need to backtest correctly and, hence, they need to follow the “real” measure, as opposed to the “risk-neutral” one. This paper shows evidence that the credit spreads of individual corporate names, by themselves, are not mean-reverting. Our results also suggest that a mean-reversion feature should be implemented in the context of joint spread-default modelling, but not in a spread-only model.

Arguably, building a credit model for counterparty risk has some important challenges. There are a number of well-developed and sophisticated models for this asset class in the literature, but they have mostly been developed in the context of derivatives pricing. A pricing model is good when it provides (i) no arbitrage gaps, (ii) good greeks that enable hedging strategies and, obviously, (iii) reasonable prices. In the world of risk management and capital calculation, a model is good when it backtests properly. That is, risk management needs the distributions arising from the models to be as close as possible to reality.

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possible to the actual distributions seen in the market. For that reason, while derivatives pricing models typically use the “risk-neutral” measure, risk management and capital calculation models tend to use the “real” measure.

In particular, in the credit asset class, one of the stylised facts that credit spread models often mimic is mean-reversion. However, there is some controversy as to whether the credit spreads are mean-reverting or not. In early studies of credit spreads, they were considered mean-reverting, but this was not the case in later studies.[1, 2].

This paper tackles that question: are credit spreads mean-reverting?

Assessing Mean-Reversion

In order to answer that question, we calibrate a number of times series of credit spreads to both an Ornstein-Uhlenbeck (OU) process

\[ dx_t = \theta_{ou}(\mu_{ou} - x_t)dt + \sigma_{ou}dW_t \] (1)

and a Black-Karasinski (BK) process

\[ d \log(x_t) = \theta_{bk}(\log(\mu_{bk}) - \log(x_t))dt + \sigma_{bk}dW_t. \] (2)

The methodology used for the calibration of these models was an Ordinary Least Squares (OLS) regression.

The authors used time series data of 5-year spreads from January 2005 to January 2012 for 23 corporate names covering a range of credit qualities. Figure 1 shows those time series.

In order to assess the validity of the measured mean-reverting parameters, the authors did two tests:

1. A Null Hypothesis Test.
2. A GBM Compatibility Test.

Null Hypothesis Test

Figures 2 and 3 show the mean-reversion speed (\( \theta \)) calibrated to those time series, both for an OU and a BK process, together with error bars\(^1\). The graphs also include the

\(^1\)Error bars indicate the 99% confidence interval assuming a normal distribution with the empirical standard deviation.
mean-reversion speed obtained from the VIX time series, for comparison, as the VIX time series is expected to show mean-reversion towards a VIX value of around 20%\(^2\).

![Image of time series graph](image1.png)

**Figure 1:** Times series of 5-year credit spreads for a number of names, from January 2005 to January 2012. Source: Credit Suisse.

Let’s say that the null hypothesis is that “the mean reversion parameter speed \(\theta\) is zero for corporate credit spreads”. The data shown in Figures 2 and 3 illustrates that this null hypothesis should not be rejected\(^3\).

![Image of mean-reversion speed graph](image2.png)

**Figure 2:** Mean-reversion Speed of 5-year credit spreads for an OU process.

The reader can see in those graphs how the zero value is included within the error bars in most cases.

\(^2\)See VIX time series in Appendix.

\(^3\)Strictly speaking, we should expect that in a few out of the 23 measurements, the zero value for \(\theta\) should be outside of the 99% confidence interval even when the null hypothesis is not rejected. However,
GBM Compatibility Test

The authors did a further test which, in their view, provides a more intuitive insight into the problem.

They generated 10,000 simulated paths using a Geometric Brownian Motion (GBM)\(^4\) process and, then, they measured the mean-reversion speed with the same procedure as used for the real data. If those GBM paths were infinitely long, then the mean-reversion speed calibrated out of them should be zero, but the GBM paths were identical in length and granularity to the credit spread time series (7 years, daily data) and, so, the calibrating algorithm will deliver non-zero values for \(\theta\) from each of the GBM paths, even when we know that they are by construction non mean-reverting. From those 10,000 \(\theta\)'s, the authors constructed statistics regarding the values of the mean-reversion speed compatible a non mean-reverting process sample (the GBM process in this case).

The distribution of those “artificial” values of \(\theta\) are shown in Figures 4 and 5 for an OU and a BK process respectively. They are shown together with the values obtained from the time series of the credit spreads (green dots) and, for comparison purposes, with the value obtained from the time series of the VIX index (red dot).

Those two figures show how the mean-reversion speed obtained from the credit spread time series are completely compatible with a GBM non mean-reverting process. This is in contrast with the \(\theta\) obtained from VIX, which is well outside the values compatible with a GBM process\(^5\).

\(^4\)With a volatility of 100%, which is a typical volatility observed in the time series of the credit spread time series.

\(^5\)Again, strictly speaking, this exercise should be done per measured \(\theta\). Ideally, the following should
Figure 4: Distribution of mean-reversion speed calibrated to an OU process in 10,000 artificially generated GBM paths (blue), together with the mean-reversion speed values calibrated from the credit spread time series (green) and the VIX time series (red).

Credit Spreads are not Mean-Reverting, at least in the “classical” way

The authors have calibrated the mean-reversion speed of an OU and a BK process to the time series of 5-year credit spreads of 23 different corporate names covering a range of credit standings, and they have shown that (i) the error of those values makes it unreasonable to reject the null hypothesis and (ii) those values seem to be compatible with a GBM process, which is non mean-reverting.

In their view, this clearly indicates that credit spreads of individual names should not be modeled with a mean-reverting process in a “classic” way. By “classic” it is meant via an OU, BK or a similar process.

However, having said that, it appears evident from market observation that credit spreads display some sort of mean-reversion feature subject to survival. This comes from the fact that, should a corporation have a credit spread of, say, 8,000 bps, we would expect it to either default soon or, if it survives in the long run, drift towards a much lower credit spread. So, mean-reversion should be captured by credit spreads subject

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be done: (i) measure $\theta$ for each name and model (as it has been done) as well as the volatility $\sigma$, (ii) generate 10,000 paths for each one of the 23 time series and model (OU and BK) with $\theta = 0$ and with a volatility of $\sigma$ and (iii) calculate a p-value of each $\theta$ from which the validity of each mean-reversion speed can be assessed. Instead, the authors have taken an average volatility of 100%, generated 10,000 GBM paths (as the credit spreads cannot be negative) and use these statistics to validate all values of $\theta$. However, the conclusion using this shortcut are so clear that the authors think that the strict analysis explained in this footnote is not strictly necessary.

$^6$Null hypothesis: “the mean reversion parameter speed $\theta$ is zero for corporate credit spreads”.
Figure 5: Distribution of mean-reversion speed calibrated to an BK process in 10,000 artificially generated GBM paths (blue), together with the mean-reversion speed values calibrated from the credit spreads time series (green) and the VIX time series (red).  

...to survival, not by credit spreads by themselves. As a result, a credit model capturing this mean-reversion must be a joint spread-default process so that, loosely speaking, the source of the mean-reversion is not in the spread evolution itself, but rather in the dependency between spread evolution and default events.
Appendix

Figure 6: VIX index time series. Source: Bloomberg.

References
