FVA Calculation and Management

CVA, DVA, FVA and their interaction (Part II)

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The calculation and management of funding risk for a portfolio of OTC derivatives is anything but trivial. In the first part of this paper (FVA Demystified), we discussed the ideas underlying FVA. We saw that it is an adjustment made to the price of a portfolio of derivatives to account for the future funding cost an institution might face. We also saw that it is very important to differentiate between the Price of a derivative (the amount of money we would get if we sell the derivative) and the Value to Me (the Price minus my cost of manufacturing the derivative). In this paper, we are going to investigate the practicalities of FVA. We will see how FVA can be calculated and managed. If we have a proper CVA system, calculating FVA is not too difficult, subject to a few reasonable assumptions. This can be achieved because a good CVA Monte Carlo simulation already calculates many of the inputs needed to compute FVA. Also, we will explain the role of FVA desks in current large organisations, as well as how FVA can be risk-managed and hedged. Finally, we propose a management set up for CVA and FVA, and understand why a number of institutions have decided to join both desks.

In the paper titled *FVA Demystified* [8] we saw what FVA is and what adjustment the

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market price of a derivative, or portfolio of derivatives, needs to account for the 'manufacturing' funding cost that we face when managing and hedging the contract.

By hedging we refer to the P&L synthetic recreation that a dealer puts in place; what it does is to recreate, as closely as possible, the daily P&L of the derivative with a symmetric P&L from the hedges, so that the overall book's[1] P&L is as insensitive as possible to changes in the market value of risk factors, like interest rates, FX rates, etc.

The source of this funding cost (or benefit) is two-fold. Firstly, there is a potential funding requirements to cover the cash misalignments between the collateral requirements in the hedging side of a portfolio (typically with exchanges, subject to an initial margin plus daily full value variation margin) compared to the derivatives side (typically bilateral agreements, with daily, weekly, etc margin calls, thresholds, minimum transfer amounts, break-up clauses, etc). Secondly, the portfolio of derivatives itself, regardless of any collateral, will have funding needs coming from premium and coupon payments. Given that banks cannot borrow any more at the risk-free rate, this cost can be important.

As a result of all this, FVA is an adjustment that we do to the market price of a derivative to account for its future funding cost.

With this in mind, we have two quantities that are important: *Price*, and *Value to Me*, where

$$\boxed{\text{Value to Me = Price - Manufacturing Cost}}$$

1. **Price**

   This is the market's price. This number reflects the cost of hedging the derivative in the 'risk-free' world that we see. It is given by two components: (i) the cost of hedging without defaults plus ($P_{\text{risk-free}}$) (ii) the cost of hedging defaults (CVA):

$$P = P_{\text{risk-free}} - CVA \tag{1}$$

2. **Value to Me**

   As seen previously, in order to calculate how much we expect to make out of a project (an OTC derivative for a dealer), we need to calculate how much the deal is worth to us, and for that we need to account for the deal's hedging (manufacturing) cost. In a bank, a major source of cost is funding. Hence, in this context,

$$V = P - FVA \tag{2}$$

   If $V > 0$, we'll make money, but if $V < 0$, it is uneconomical for us to enter the deal.

[1] OTC derivatives plus hedging positions.

A Healthy Disclaimer As said in the first part of this paper, we would like to be able to say that we know everything about FVA, and that we understand it perfectly. However, we must admit that is not the case. FVA is a complex matter that is still under discussion in the industry and academia. As a result, we need to be humble and accept that the best we can do in this paper is explain our view. This view is the result of a lot of research, conversations, and thinking, but is always developing. Because of this, we must be realistic and accept that we may change our mind about FVA in the future.

We think this is the only reasonable approach to this question.

Analytical Black-Scholes Pricing with Collateral and Funding

There have been a number of attempts to calculate the price of a derivative when it is collateralised and faces non risk-free funding costs, from a theoretical standpoint. This has been done by modifying the Back-Scholes risk-neutral pricing framework to account (i) for borrowing and lending rates that are different to the risk-free rate and (ii) for collateral that is posted and received under the bilateral CSA agreements [4, 7, 6, 1, 5].

In a nutshell, the standard Black-Scholes framework defines the operator

$$L = \frac{\partial}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2}{\partial S^2}, \tag{3}$$

where S is the underlying risk factor, and states that the change in value V over time for a risk-free derivative is given by the differential equation

$$L \cdot V_t = r\, V_t - r\, S\, \frac{\partial V_t}{\partial S}, \tag{4}$$

where r is the ‘risk-free’ rate at which the dealer can borrow and lend in unlimited quantity, as often as needed, without trading costs. Also, the dealer can buy, sell, borrow and lend the underlying risk factor S in unlimited amounts, as often as needed, without trading costs as well.

Importantly, by ‘risk-free’ it is meant that the resulting portfolio of the derivative, plus any cash borrow or lent, plus any asset S bought or sold, is market-risk-free for the dealer. Another important assumption is that defaults do not exist: all parties will honour their financial commitments, with certainty.

Using Peterbarg’s version of the Black-Scholes expansion for collateral and funfing [7], the new equation that sets the evolution of the V is

$$L \cdot V_t = r_c\, C_t + r_f\, (V_t - C_t) + (r_d - r_r)\, S\, \frac{\partial V_t}{\partial S} \tag{5}$$

where r_c is the rate returned on collateral under the CSA agreement, r_f is the unsecured funding rate for the derivative dealer, r_r is the secured (repo) funding rate and r_d is the rate of return on asset S.

These approaches also make the usual assumptions; that the posting and receiving of collateral is continuous and free, and that borrowing, lending, buying and selling are possible continuously and free of any cost.

Key Elements of a FVA Calculation

The analytical pricing efforts described above deal with the case when, under the Black-Scholes framework, an institution can only borrow cash at a risky rate, which is different to a market's 'risk-free' rate. Also, it deals with the collateral from the bilateral agreement with the counterparty. In that sense, the modified Black-Scholes framework is an important step forward in quantitative finance.

However, we may not be able to use it to calculate FVA. That is because there are a number of important key features that need to be accounted for.

- **Bilateral CSA agreements can be quite complicated**

  The modified Black-Scholes framework still relies on some highly idealistic assumptions regarding collateral arrangements with the counterparty. Those equations can only be solved for the special cases when a portfolio is either fully collateralised ($C_t = V_t$), or with no collateral at all ($C_t = 0$). However, bilateral CSA agreements can be quite complicated, with thresholds, lack of symmetry, multi-currency features, break clauses etc.

- **A particularly important source of FVA is in the collateral misalignment with the hedging institution**

  Also, the modified Black-Scholes framework does not touch on the effect of collateral non-alignment between the bilateral CSA and (typically) the exchange where the hedging positions lie. In fact, this is one of the most important sources of funding risk. This is depicted in Figure 1. It can be seen here that the dealer has a net loss coming from the funding requirements for the difference in collateral needs in each side of the deal. FVA in this context is the expected present value of that loss or gain.

- **Institutions' borrowing and lending rates are not symmetric**

  Another idealistic assumption is that financial institutions borrowing and lending rates are the same. This is not the case in real life.

- **FVA must be calculated at portfolio level, while CVA is a netting set level number**

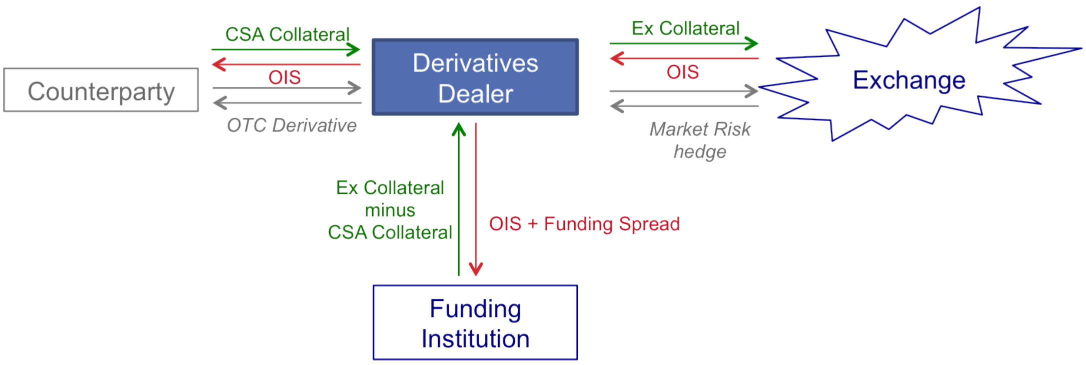

Figure 1: Illustration of the source of funding cost for an collateralized OTC derivative.

Figure 2 illustrates the point that FVA must be calculated at portfolio level. Let's say that we have two different counterparties, and we have the same trade with both of them, but we are long in one, and short with the other one. Each trade is a netting set by itself, and both netting sets have the same CSA agreement, which is symmetric between the dealer and each counterparty.

Figure 2: Illustration of portfolio netting effects for FVA.

In this case, each netting set will have its own CSA calculation, but any collateral that needs to be posted by one counterparty will be equal to that needed, in the opposite direction, by the other counterparty. So, as long as rehypotication is allowed, this setup has no funding requirements at all.

The only way to compute a zero FVA in this case is if we calculate that number at portfolio level. By portfolio we mean the whole book of derivatives that a financial institution has or, going even further, all the financial positions the institution has, if applicable.

The FVA calculation - a Monte Carlo simulation

It appears that the only way we can compute FVA in a realistic way is via a complete Monte Carlo simulation. This is the case because analytical attempts at FVA are too idealistic.

We might argue that an ideal solution to this problem is introduced by Dr Choudhry in his talks [2]. He argues that a derivative is no more than a set of future cashflows. Hence, if future cashflows in an institution are valued using its funding rate, the same should happen with derivatives. We agree this could be an perfect solution, but trying to simulate *all* cashflows an institution is expecting in the future through a Monte Carlo simulation, seems essentially impossible right now from a technical standpoint.

A practical solution

Be to realistic, we need to compromise and find good proxies to make the FVA calculation feasible. We are going to see how we cancompute FVA in stages.

The two sides of FVA

In principle, FVA has two components, one that accounts for the cost of borrowing collateral, the Funding Cost Adjustment (FCA), and one accounting for the benefit of lending excess collateral, the Funding Benefit Adjustment (FBA).

$$FVA = FCA + FBA \tag{6}$$

Let's say that s_t^{borrow} and s_t^{lend} are the borrowing and lending spreads over IOS that an institution faces[2]. Let's define

$$EPE_t^{cash} = \mathbb{E}\left(\max\left(C_t, 0\right)\right) \tag{7}$$

$$ENE_t^{cash} = \mathbb{E}\left(\min\left(C_t, 0\right)\right) \tag{8}$$

where C_t represents an institution's cash needs at the time t. FCA should reflect the present value of the expected cost of our borrowing needs, in order to meet our cash obligations; FBA should be the same, but from a lending standpoint. Hence, we define FCA and FBA at time zero (today) as

$$FCA_0 = \int_0^T EPE_t^{cash} \cdot DF_t \cdot s_t^{borrow}\, dt \tag{9}$$

$$FBA_0 = \int_0^T ENE_t^{cash} \cdot DF_t \cdot s_t^{lend}\, dt \tag{10}$$

where DF_t is the risk-free discount factor at time t, and T is the maturity of the last trade in the portfolio.

We will see later in the paper that, in several cases, the FBA component can be neglected.

[2]To be more precise, these are the unsecured forward short spreads.

The Monte Carlo simulation

In order to calculate FVA, we need to compute Equations 9 and 10. We are going to focus the explanation on Equation 9 for simplicity, but all said can be extrapolated to Equation 10 easily.

In Equation 9, DF_t and s_t are given by the market, so the only computation is EPE_t^{cash}, through Equation 7. Let's do that now.

The problem of calculating FVA is now reduced to three problems: (i) simulating the collateral needs for the counterparties with whom we have the collateral agreements, (ii) simulating the collateral needs for the exchanges where the hedging positions sit and (iii) simulating the cash flows needs that each derivative carries[3].

In this calculation, we are going to assume that all collateral is cash, and in the same currency. Later we will see how to deal with those other risky collateral cases.

Let's build this calculation step by ste.

Recycling the CVA calculation for FVA

Collateral with Counterparties In order to simulate the collateral with all counterparties, we need to do the following. To start with, we need to fix a number of time points in the future where we want to calculate the collateral, and we are going to decide on a number of scenarios for our Monte Carlo simulation. Typically, this will be around 100 time points and around 10,000 scenarios. Then:

1. **Risk Factor Evolution**

   First we need to evolve all risk factors that affect the value of the trades. By risk factors we mean yield curves, FX spot rates, equity prices, credit spreads, commodity prices, inflation curves, etc. We might easily have more than 1,000 risk factors, which means that we need to generate around 1 billion risk factor numbers!

2. **Derivative Pricing**

   Once we have all the risk factor values, we need to price each derivative contract in all the 1,000,000 time points and scenarios. A large institution is going to have around 1,000,000 contracts, which means they need to generate around 1 trillion prices!! The reader can imagine the demanding computing effort this step can take if there are exotic, or even semi-exotic, contracts in the portfolio.

3. **Derivative Pricing Netting**

[3] By this it is meant cashflows like option premiums, coupon payments, etc, not the collateral calls.

Once we have all 1,000,000 prices per derivative, they are netted per netting set. This means that the prices of each derivative are grouped by scenario and time step. On this way, we end up with one price grid[4] per netting set.

4. **Collateral Simulation**

   Collateral also needs to be simulated. This is done in each of those netting set grids, as the CSA collateral agreements apply for each netting set. We need to simulate it while considering re-margining frequency, thresholds, minimum transfer amounts, rounding, haircuts in non-cash collateral, multi-currency options, initial margin, etc.

   When this step is done, we are going to end up with one grid of collateral per netting set.

5. **Collateral Netting**

   We have seen that FVA crystallises at portfolio level, so we need to allow for this in our calculation. This can be done by summarising all collateral grids to come up with one overall portfolio collateral grid. This should contain the collateral that the institution will need in the future, in each scenario and time point.

The bad news is that it is terribly complicated to derive this portfolio collateral grid. However, the good news is that, if we have a good CVA system, most of these steps have now already been done for CVA. Steps 1, 2, 3 and 4 can be recycled from the CVA system, so that only step 5 needs to be redone now. And this step is very simple: just summing up all collateral values per scenario and time step.

If we denote by i each scenario, and by t_j each calculation time point[5], then

$$C_{i,t_j}^{CSA} = \sum_k C_{i,t_j}^{CSA,k} \tag{11}$$

where k counts through all the netting sets.

It must be noted that, in this method, any netting set that is not collateralised is accounted for automatically here, as in those cases all we have to do is set $C_{i,t_j}^{CSA,k} = 0$.

However, we need to make a special distinction regarding rehypothication. We have said before that CSA that does not allow for rehypotication, and has no practical effect in CVA. However, it does have effect in FVA, as the collateral received by the dealer cannot be posted to the exchange where the hedging sits. This can be implemented in the calculation quite simply by setting $C_{i,t_j}^{CSA,k} = 0$, for FVA calculation purposes, in those netting sets where rehypotication is not allowed.

[4] One grid is each collection 1,000,000 values, coming from the 100 time steps × 10,000 scenarios.

[5] That is, i ranges from 1 to 10,000, and j from 0 to 100.

Collateral with Exchanges The other aspect we need to consider is the collateral posted (or received) to the exchanges where the hedging positions are. This collateral comprises two parts: initial margin (IM) and variation margin (VM).

$$C_t^{hedge} = IM_t^{hedge} + VM_t^{hedge} \tag{12}$$

Let's start with the easier of them: variation margin. Exchanges operate under fully collateralised conditions, so the collateral that needs to be posted is going to be the value of the whole portfolio of derivatives.

If we denote by P_{i,t_j}^l the simulated price[6] of trade l in scenario i and time step t_j, then the grid of variation margin values will be given by

$$VM_{i,t_j}^{hedge} = \sum_l P_{i,t_j}^l \tag{13}$$

Again, the good news here is that those P_{i,t_j}^l grids have already been calculated in the 'derivative pricing' step of the CVA calculation, hence calculating the variation margin grid is quite a trivial computation. It must be noted that, if we prefer, we could sum up P from each netting set, as the result is going to be the same as summing up all individual trades.

Having done that, now we are left with the calculation of the IM requested by the exchanges. This bit is somewhat more tricky, as the CVA calculation does not consider how the portfolios are hedged, and hence we cannot recycle any existing computation. There are a number of approaches that we could take here.

1. The first approach consists in replicating the IM calculation that the exchanges do. Assuming that exchanges base initial margin in 10-day 99% VaR, under normal conditions, quite a typical metric, in each time step we can say that

$$IM_{i,t_j}^{hedge} = 2.33\sqrt{\frac{10}{260}}\,\Omega \tag{14}$$

   where Ω is the annual volatility of the portfolio in our simulation. Calculating Ω may turn out to be difficult, but not impossible. We need to know the delta of each trade to each risk factor, in each scenario, then sum all the deltas of all trades, multiply them by the volatility of each risk factor and then add the variances of each of those sensitivities to derive to the final portfolio volatility. In this way, we will come up with a grid of initial margin IM_{i,t_j}^{hedge}.

   However, this could be quite problematic given its calculative difficulty, and also because, at the end of the day, we are guessing the methodology used by the exchanges for the IM calculation.

[6] Under the Black-Scholes risk-neutral pricing framework.

Alternatively we could ask the exchanges what methodology they use, but there is no reason why the exchanges will not change it in the future, and hence we are going to have to notice that change first, and then modify our implementation.

2. Another approach could be to do a historical analysis and find patterns that we could easily use. For example, if we are a large dealer, with massive positions in exchanges, we would not be surprised if we found a relationship along the lines of $IM_t^{hedge} = \alpha \cdot P_t$, where P is the value of the positions we hold with the exchange and α is a constant to calibrate historically. Also, we could test a hypothesis along the lines of $IM_t^{hedge} = \beta \cdot \Omega_t^{\gamma}$, where Ω is the volatility of the portfolio and β and γ are constants, or any other law we find to be suitable.

   The key idea here is to do a historical analysis to find a (hopefully) easy statistical relationship $IM_t^{hedge} = f(X_t)$, where X_t is something we already simulate in the calculation.

Ideally, we want to find that

$$IM_t^{hedge} = \alpha \cdot P_t, \tag{15}$$

because the equations simplify nicely if this holds. When this is the case, the collateral held with the exchanges is quite simple to calculate. It is given by

$$C_{i,t_j}^{hedge} = (1+\alpha) \sum_l P_{i,t_j}^l \tag{16}$$

where l ranges across all the trades in the portfolio. The '1' in this equation accounts for the variation margin, and the 'α' for the initial margin.

Cashflows from derivatives The final part of the calculation comes from the actual cash needs of the derivative, regardless of any collateral arrangements. By this we mean option premiums, coupon payments, etc.

Let's illustrate the source of this funding risk with the following example. Let's imagine we are a derivatives dealer and sell an option to a client for, say, \$1. After we do that sale, we are going to set a symmetric hedging position in an exchange; we are going to buy the same option, from an exchange, at, say \$0.9. In this deal, we make \$0.1. If we imagine now that both the bilateral agreement with the client and with the exchange operates in a uncollateralised basis, for illustration purposes, then what we are going to do is transfer \$0.9 from the payment we receive from the client to the exchange, and the other \$0.1 is going to stay in our pockets. As a result, we are going to have a funding benefit in this example: we can use that \$0.1 to reduce overall funding needs.

Obviously, there are going to be some cases where the cashflows from derivatives will provide a funding benefit, but some other times they will create a funding cost. Some institutions may decide to set these cashflows completely outside of any funding considerations, and dedicate them solely to pay running costs, salaries, dividends to shareholders, etc. However, if these cashflows are intense enough, then it may be advisable to study their funding impact, as they could create a competitive advantage, or disadvantage, for the institution.

Unfortunately, this calculation can be quite tricky. What we need to do is calculate, in each time step and scenario of the Monte Carlo simulation, (i) the actual cash flows that each derivative has experienced during the previous time step, together with (ii) the cashflows from its hedging side and, then (iii) subtract one from the other one, and calculate on this way the cash excess or shortage. Doing steps (i) and (ii) can be quite difficult, specially for exotic derivatives, as we will have to do both a cash flow and a hedging simulation; standard pricing functions may not be readily available for this and may need to be modified.

Having said that, apart from technical difficulties, what we are going to generate now is a portfolio *cash account*, to which each derivative adds or subtracts as cash flows take place. If we do this, we are going to come up with a simulated portfolio of derivatives account $C_{i,t_j}^{portfolio}$,

$$C_{i,t_j}^{portfolio} = \sum_l C_{i,t_j}^l \tag{17}$$

This cash account is going to be the sum of all *past* cashflows coming from the derivatives and their hedging positions, regardless of any collateral arrangements. It should be noted that, in general, that account may not start with a zero value.

This calculation may be important if a dealer has a business line that is highly cash intense, with strong swifts in its inflows, outflows and fee income. The author of this text knows of at least one tier-1 investment bank that has this type of business line, and that is considering implementing this calculation.

Calculating FVA

Now we have all the components needed to calculate FVA.

As a first approximation, we can say that the portfolio cash account is always zero (or very small compared to the collateral needs) as fee inflows and outflows are small compared to the collateral needs, or are used for overall fixed costs, salaries, dividends to shareholders, etc. So, if we put together Equations 7, 11 and 16 we obtain

$$EPE_t^{cash} = \mathbb{E}\left(\max\left((1+\alpha)\sum_l P_t^l - \sum_k C_t^{CSA,k}, 0\right)\right) \tag{18}$$

where P_t^l and $C_t^{CSA,k}$ have already been calculated by the CVA engine, and α is calibrated historically. With this EPE profile, we can now calculate Equation 9 quite easily:

$$FCA_0 = \int_0^T EPE_t^{cash} \cdot DF_t \cdot s_t^{borrow} \, dt$$

It must be noted that this methodology accounts for the desired features we discussed before, since:

- It computes the expected funding cost of the collateral misalignment between the derivatives and the hedging side of the portfolio of trades.
- It nets trades and collateral at the right level for FVA: at portfolio level.

However, if for some reason we want to be more precise, or we have a portfolio of derivatives that is very cash demanding, then we need to consider Equation 17 in the calculation by computing

$$EPE_t^{cash} = \mathbb{E}\left(\max\left((1+\alpha)\sum_l P_t^l - \sum_k C_t^{CSA,k} + \sum_l C_{i,t_j}^l, 0\right)\right). \tag{19}$$

For the remainder of the text, we will stick to Equation 18 for illustrative purposes.

A few special cases

Let's consider now a few special cases, so we gain some intuition as to what we are doing here and we can put all this in context.

Fully Collateralised Portfolio Let's say that our portfolio of trades is fully collateralised, with the ISDA's standard CSA[7]. In this case, the collateral needed in the CSA side equals the value of the book of trades

$$\sum_k C_t^{CSA,k} = \sum_l P_t^l \tag{20}$$

and so, Equation 18 gets reduced to

$$EPE_t^{cash} = \mathbb{E}\left(\max\left(\alpha \sum_l P_t^l, 0\right)\right) \tag{21}$$

[7]Only cash collateral for variation margin, all in the same currency, daily margining, fully collateralised, etc.

or, given that α is a constant number,

$$EPE_t^{cash} = \alpha \, \mathbb{E}\left(\max\left(\sum_l P_t^l, 0\right)\right) \tag{22}$$

It must be noted that the last term of that equation is precisely the EPE profile of the whole portfolio of trades when considered to be uncollateralised,

$$EPE_t^{portfolio,uncollateraised} = \mathbb{E}\left(\max\left(\sum_l P_t^l, 0\right)\right). \tag{23}$$

As a result,

$$EPE_t^{collateral} = \alpha \, EPE_t^{portfolio,uncollateraised} \tag{24}$$

In other words, the EPE of the collateral needed in a fully collateralised portfolio is the expected value of the initial margin posed to the exchanges, which could be approximated by α times the EPE of the portfolio when considered uncollateralised.

With this, FBA should be then quite a straightforward calculation

$$FCA_0 = \alpha \int_0^T EPE_t^{portfolio,uncollateraised} \cdot DF_t \cdot s_t^{borrow} \, dt \tag{25}$$

which rounds off the result nicely.

For simplification of notation, let's define

$$FCA_0^* = \int_0^T EPE_t^{portfolio,uncollateraised} \cdot DF_t \cdot s_t^{borrow} \, dt \tag{26}$$

which represents the FVA of a book of trades that is fully uncollateralised, and that is hedged with full collateralisation but zero initial margin. In this case,

$$FCA_0 = \alpha \, FCA_0^* \tag{27}$$

as α accounts precisely for the initial margin.

Fully Uncollateralised Portfolio Let's say now that we are in the opposite case, where our portfolio of trades is fully uncollateralised. In that case,

$$\sum_k C_t^{CSA,k} = 0 \tag{28}$$

and, then,

$$\begin{aligned} EPE_t^{collateral} &= \mathbb{E}\left(\max\left((1+\alpha)\sum_l P_t^l, 0\right)\right) && (29)\\ &= (1+\alpha)\,\mathbb{E}\left(\max\left(\sum_l P_t^l, 0\right)\right) && (30)\\ &= (1+\alpha)\,EPE_t^{portfolio,uncollateraised} && (31)\end{aligned}$$

As a result,

$$FCA_0 = (1+\alpha)\,FCA_0^*. \tag{32}$$

Partially Collateralised Portfolio In reality, some of the trades will be subject to CSA agreements, and some will not. If a γ proportion of the portfolio is collateralised, then

$$FCA_0 = (1+\alpha-\gamma)\,FCA_0^*. \tag{33}$$

The case of a CCP There is a strong regulatory push to clear all OTC derivative trades through central counterparties (CCP). When that happens, counterparty risk is supposed to be reduced[8]. However, this is not obtained for free, as the counterparties now face the funding cost of setting this up.

For example, let's say that we are a dealer. We have seen that if the whole portfolio of trades is under bilateral agreements, and fully collateralised, then $FVA_0 = \alpha\, FVA_0^*$. If we novate all trades to a CCP, then the variation margin asked by one side (ie. the hedging side) will be the same as that delivered by the other side (ie. the CCP) so there is no funding risk coming from variation margin requirements. However, we have to post initial margin on both sides, and so we have a funding hit from both sides. If the initial margin posted to the CCP can be expressed as $IM_t^{CCP} = \beta \cdot P_t$, then the FVA of this setup can be expressed as

[8] We say that it is *supposed* to be reduced because it is far from clear that this ambition is attained. We identify three reasons. Firstly, global exposure is supposed to be reduced via the multi-lateral netting that takes place in the CCPs, but some academic studies show that, in practice, the opposite may happen [3]. Secondly, the default probability attached to a deal decreases, as CCPs are built to be super-AAA entities. However, the exposure subject to default, which is what we care about in counterparty risk, can be massive, as a CCP defaulting would cause absolute chaos in the financial system. Because of this, in our view CCPs do not really reduce counterparty risk, they only reshape it, as they actually create tail default risk.

$$FCA_0 = (\alpha + \beta)\, FCA_0^*. \tag{34}$$

Personalising FVA

We have seen in this text, as well as in its Part I sister [8], that FVA should not be seen as a a price, rather as a risk reserve. That is because it reflects the funding 'cost of manufacturing', which is intrinsic to each institution. Because of this, the FVA calculation should be a reflection of the true sources of funding risk an organisation faces.

We have already seen an example of this, with the 'cashflows from derivatives' side of the calculation, that comes from fees income. Some institutions may decide to put this cash to ease funding needs, some may not. The FVA calculation should reflect so, accordingly.

Another most important source of differentiation is in the way the FVA desk operates internally. There are two typical cases, that we are going to review here.

The case of a small outlet

If we are a small outlet, the desk that deals with FVA and that provides funding to the dealing desks may be the same desk that goes out in the market to borrow from investors, and also to lend out any excess in cash. That desk is going to face an asymmetric funding spread. It is going to borrow at a given credit spread s^{borrow}, but any cash excess cannot be lent out without risk at any rate other than the risk-free rate, which means that its lending spread s_t^{lend} will generally be zero. Another way of seeing this is that if the institution decides to lend cash out, above the risk-free rate, it will then be taking credit risk, which would need to be accounted for, typically by discounting those loans at a risky discount factor.

All this leads to the fact that, in practice, in this case, FBA could be ignored, is funding benefit cannot be practically exercised without generating other risks.

Because of this, in practice, we can say that,

$$\begin{aligned} FBA &\simeq 0 \\ FVA &\simeq FCA \end{aligned} \tag{35}$$

and we can ignore the funding benefit side of the problem.

The case of a universal bank

A universal bank is going to be right the opposite case. These kine of institutions have, typically, three business lines: investment banking, retail banking and private banking. The heavy derivatives business lies within the investment banking unit.

Typically, an organisation is going to have a treasury desk, in charge of borrowing funds from external investors to, subsequently, lend them out to its units. From a derivatives standpoint, the dealer is going to have a FVA desk, that provides funding to all the dealing desks. This FVA desk is going to borrow from the treasury desk. Also, importantly, it is going to lend excess cash to the treasury, so other part of the organisation are benefited by it. In other to set up this correctly, with the right incentives, treasury should pay the FVA desk a lending rate for the cash it is borrowing from it. Generally, that lending rate is the same as the borrowing rate it charges. On this way, the derivatives business is going to enjoy the benefits of good funding management, and the lack of symmetry referred before is left to treasury to deal with, at global corporate level.

This has consequences for the FVA calculation as, now, $s_t^{borrow} = s_t^{lend} = s_t$, and given that

$$MtM_t = EPE_t + ENE_t \tag{36}$$

then,

$$FVA_0 = \int_0^T MtM_t^{cash} \cdot DF_t \cdot s_t \, dt \tag{37}$$

As we are going to see later, which of these two cases an organisation operates in is also going to determine how to manage funding risk. Sometimes they are referred to as symmetric (universal bank) or asymmetric (small outlet) funding risk.

Fine-Tuning the Calculation

There are a few other considerations that we may want to take into account.

Risky collateral

So far, we have assumed that the collateral is in cash, and in the same currency as the portfolio of trades. There is a push in the industry to migrate CSA agreement to the recently published ISDA's standard CSA, as this problem is mostly avoided with them[9]. However, there will surely still be cases in which bespoke CSAs are used, amongst other reasons because cash collateral is not always the least risky option.

Typical non-cash collateral is highly rated sovereign bonds, but other securities like corporate bonds or equities can be posted as well. Also, when cash is posted as collateral, but in a different currency to that of the portfolio of trades, then the whole CSA facility (ie. portfolio of trades plus the collateral) has FX risk, hence this cash also constitutes risky collateral.

A typical way to deal with this is by applying a haircut to the collateral so that, for example, $100 of collateral nets only, say, $95 of exposure (a 5% haircut). This haircut is supposed to account for a typically 10-day move in the price of the collateral, should the counterparty default. The higher the risk of the collateral, the higher the haircut.

However, this overestimates collateral needs and will lead to a miscalculation of FVA. This is because the dependence between the value of the portfolio and the value of the collateral is not considered. A good illustrative example of this is if we have a netting set composed on one long equity forward. In this case, the least risky collateral that we can receive is, precisely, the equity the forward is referring to. This is so because, in that case, the value of the trade will move in parallel with the value of the collateral, hence showing quasi-zero risk in the CSA facility[10]. Because of this, in order to assess the riskiness of a collateral we have to also have a look at the trades in the netting set it is attached to.

The optimal way to calculate FVA when risky collateral is in the netting set is simulating the collateral in the Monte Carlo simulation. In this way, Equation 11 will contain that simulation, and this dependency we are referring to will be captured naturally in Equation 7[11].

Secured V. Unsecured Borrowing

Regardless of how we calculate the risk given by risky collateral, receiving such collaterally should, generally, decrease FVA by comparison with not receiving any collateral at all. This holds because, if we can rehypothicate the risky collateral, then we can always borrow cash in a secured transaction, via the repo or the forward FX market. The repo and FX spreads will generally be very small compared to our unsecured funding spread. This is illustrated in Figure 3.

In this case, the calculation of Equations 9 and 10 needs to be split into two, one that accounts for secured and another for unsecured borrowing.

[9] But not completely, as initial margin can still be posted with highly rated bonds.

[10] Strictly speaking, there could still be some residual interest rate risk, equity-repo risk or dividend risk.

[11] The simulation of the collateral needs to be done with the appropriate dependency structure with all other risk factors that affect the risk-neutral price of the trades in the netting set.

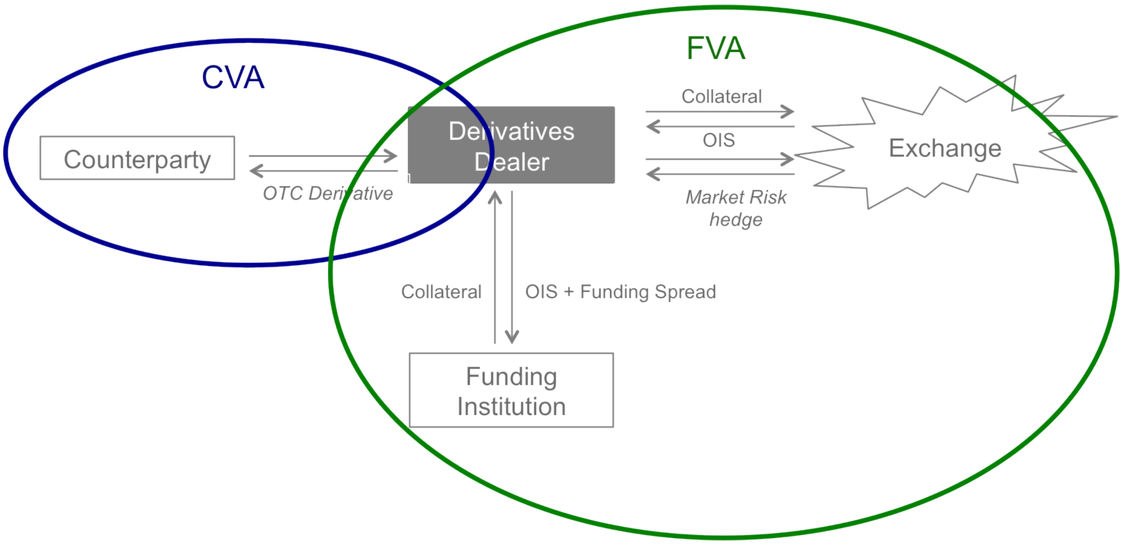

Figure 3: Illustration of the source of funding cost when secured and unsecured borrowing takes place.

It must be noted that in many cases, like for AAA government bonds, the repo spread is so small that we may want to ignore it in the calculation if it complicates things more than the benefit we gain. Also, those bonds can generally be posted as initial margin, so no repo transaction is needed for them.

Settlement Risk

So far the whole discussion is based on the fact that any collateral we receive can be posted as collateral 'instantaneously'. This is obviously not the case. In reality, we are typically going to have a 1-day settlement lag in this process. As a result, we may have to fund, for one day, all the collateral needed to be posted and that we have not actually received yet.

A finer calculation should also account for this. In order to do so, the collateral algorithm will have to consider the collateral that is on the way to us but has not arrived yet, and discount it from today's net theoretical collateral position, to come up to the true collateral that we need to fund.

It must be said that in some business lines, such as when a CCP member offers clearing services to clients that non-CCP members, the effect of this settlement lag can be quite important.

Funding with Right and Wrong Way Risk

Also, up to now we have implicitly assumed that the portfolio of trades under consideration does not fund any right or wrong way risk. However, sometimes we may have this kind of risk in funding. This is typically going to happen when we have credit trades in the portfolio with obligors that are highly related to us. For example, if we are UBS, and we have in our portfolio CDSs with Credit Suisse as the obligor, then there is going to be a dependency between the value of that CDS and our funding spread. Having said that, given that FVA is a global portfolio calculation, this effect should be generally small.

In any case, when we have relevant right or wrong way risk, we have to take one step back in the calculation and solve

$$FCA_0 = \mathbb{E}\left(\int_0^T \max\left(C_t, 0\right) \cdot DF_t \cdot s_t^{borrow}\, dt\right). \tag{38}$$

and similarly for FBA_0.

There are a number of papers in the literature that deal with right and wrong way risk. The reader could be referred to *Optimal Right and Wrong Way Risk* [9], where the authors show a review of available methodologies.

Managing Funding Risk

So far we have discussed how to calculate FVA, but not how to manage its associated risk. We discuss that now.

We have already seen that FVA is not a price; rather, a risk reserve. When marking the price of derivatives to the market, for balance sheet purposes, CVA is accounted for, with both its asset and liability side, as it reflects a true profit or loss that can be realised when a derivative is transacted. However, FVA is an internal calculation to account for funding the 'manufacturing' cost, so we make sure we do not loose money in a contract that would otherwise appear profitable.

A typical way to handle this is via a FVA desk. This desk will be in charge of providing all funding the dealing desks need, so they (the dealing desks) can forget about that problem. For this service, the FVA desk is going to charge, to each dealing desk, at trade inception, how much it is expected to spend (or gain) in funding as a result of the new trade. The same method is applied if a trade is unwound.

This has a number of important consequences:

- In the CVA world, the CVA desks are in charge of hedging both (i) default events and (ii) undesired CVA fluctuations in the balance sheet, as much as possible given market restrictions. Within the important practical trading constrains, a CVA desk could hedge default events by rolling short-term CDS positions with a notional equal to the current exposure, but this leaves CVA volatility naked. Also, it can choose to hedge only CVA volatility by setting a number of CDS positions in credit indices, for example, that will minimise CVA volatility, but offer no protection at all to actual counterparty default events. In practice, CVA desks tend to use a blend of those two strategies, as far as permitted by trading market conditions.

However, the balance sheet does not have any FVA component, and so it is not affected by FVA volatility. As a result, FVA desks need to focus only on hedging actual funding risk, and they can forget about FVA volatility as such.

- So far we have discussed how to calculate FVA, but the reader must note that the number that is usually most relevant is the *incremental* FVA that a new trade generates, or that the unwinding of an existing trade brings along. That is the number that should be charged (if positive) or given (if negative) to a dealing desk for each operation they do. On this way, the dealing desks will be directly sensitive to the marginal funding cost they create with their activity, and so their incentives are aligned with the institutional ones.

- Given that FVA is not a price, but a risk metric, the FVA number should reflect the actual funding risk management strategy that the organisation has, or wants to have. For example, given the funding liquidity tightness that banks have suffered in the recent past, a bank may decide to be 'conservative' when calculating FVA and define it as, say,

$$FCA_0 = \int_0^T \max(EPE_t^{cash}, PFE60_t^{cash}) \cdot DF_t \cdot s_t^{borrow}\, dt \tag{39}$$

  where $PFE60$ refers to the Potential Future Exposure profile at 60% confidence level. With this kind of FVA metric, the FVA desk will be overcharging with respect the typical FVA calculation, but this may make sense to account for the extra funding liquidity cost the funding desk actually faces, rather than the ideal assumptions taken in the standard FVA calculation. Another way of doing this could be, for example, with

$$FCA_0 = \int_0^T EPE_t^{cash} \cdot DF_t \cdot (s_t^{borrow} + \delta s)\, dt \tag{40}$$

  where δs is an extra spread applied to account for mismatch between the ideal FVA assumptions and the reality of managing funding risk.

  By using this kind of approach, the institution will be constraining business development for the sake of decreasing funding risk. In the next section we are going to see the types of liquidity risk that this overcharge intends to cover.

Funding Liquidity Risk

As just indicated, one of the key challenges of financial institutions is how to manage funding risk. In particular, how to manage its liquidity, given the major constraints they have faced in that area in recent years. There are two fundamental ways we can manage funding liquidity risk:

1. On one hand, theoretically, to be fully hedged from a funding perspective, a institution should borrow forward its expected funding needs, with a notional profile that follows the EPE_t^{cash} or the MtM_t^{cash} profile, depending on the set up as explained before. This is described in Figure 4. Also, it should hedge the sensitivities of the chosen profile to market risk factors like interest rates, FX rates, equity prices, commodity prices, etc. In this way, any P&L that comes from these position will fund (again, in theory) any extra funding needs coming as a result of movements in the market.

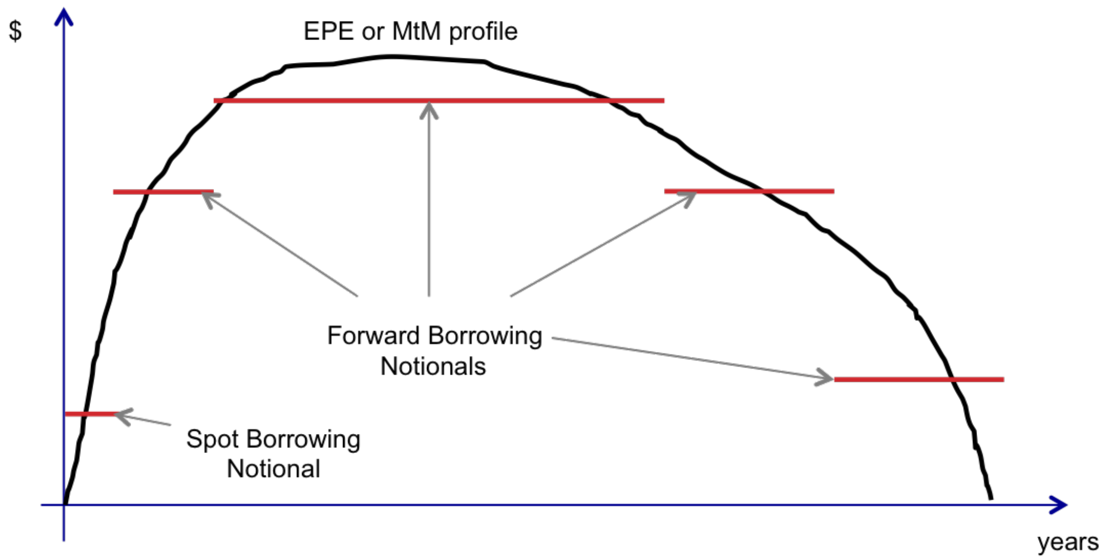

Figure 4: Illustration of the notional profile needed to hedge funding risk.

   Unfortunately, when we face reality, things may not be as simple as that. Borrowing forward, in any amount, whenever is needed, is not that simple, specially if you are a treasury facing the 'outside' market to borrow from[12].

2. On the other hand, an alternative option would be simply not to worry at all about future forward funding needs, and survive from short term borrowing. An extreme case of this strategy would be to borrow every night, all the funding required

[12] As opposed to an FVA desk, facing an internal treasury desk to borrow from, where there may be more flexibility.

for that day. Obviously, this is very risky, as if one day the market does not have enough liquidity, we could easily run into serious problems and, eventually, default. Also, our funding cost will be highly exposed to movements in our funding rate. For these reasons, this type of on-the-go funding strategy is very risky.

As the reader would expect, the solution to this problem is to find the right balance between the funding cost and the funding risk the institution is willing to take. That is, to find the optimal middle point between the two strategies described. A typical solution could be building a short, medium and long-term borrowing profile, so that we borrow long term a chunk of the expected funding needs that are rarely touched, then build medium term positions that are managed periodically, and a final short term 'top up' borrowing block that is managed daily.

Given the complexity of funding, sophisticated financial institutions now have a FVA specialised desk to manage this risk. The key role of this desk is to maintain a realistic view of what can and cannot be done in the real funding market place, design a strategy that finds the desired balance between cost and risk, implements it and charges the dealing desks for the FVA amount that is needed to subsidise that strategy. In the case of a 'universal bank' model described, the FVA desk needs to work very closely with the treasury desk.

It is very important for a bank that short term cash reserves[13] are enough to withstand the market liquidity squeeze as designed in the funding strategy. In fact, as a result of the 2007 funding squeeze, Basel III created two funding ratios: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). The LCR requires a bank to hold sufficient high-quality liquid assets to cover its total net cash outflows over 30 days. The NSFR seeks to provide enough funding liquidity over a one year period.

Conclusions (of this Part II paper)

We have seen how we can reuse the Monte Carlo simulations that we may have for CVA, for this FVA computation. In fact, if we have a proper dynamic collateral model for CVA, calculating FVA requires

1. Adding a number of previously calculated grids (Equations 11 and 13),
2. Estimating the future initial margin that will be required from us (Equation 15),
3. If needed, simulating a cash account for derivative premiums and coupons,
4. Calculating an EPE profile (Equation 18), and
5. Computing numerically a simple integral (Equation 9).

[13] This includes the so-considered 'cash equivalents', like US government debt securities.

The only steps which might cause difficulties are 2 and 3. Step 2 may be solved with some time series analysis, and step 3 may only be required for highly cash intense businesses.

Also, we have seen that managing funding liquidity risk is of critical importance to financial institutions at present. The 'ideal' hedging strategy is impossible to attain in real markets, so one the key roles of FVA desks is to have a good understanding of what can and cannot be achieved, implementing the desired funding strategy and creating the correct funding incentives in the organisation, that reflect that strategy.

Final Conclusions (of Part I & II of this paper)

This paper, together with its part I [8], explained the concept, calculation practicalities and risk management issues around FVA, and how it interconnects with both the asset and liability sides of CVA.

The main takeaway from these papers is that CVA and FVA are related in their calculation, but each is attached to a very different concept. CVA is a number used to calculate the market price of a netting set, while FVA is a number used to calculate the 'manufacturing' cost, at portfolio level.

FVA must not be used to calculate the *Fair Price* of a deal, as that price is our best estimate for the exit price, which is set by the market. That price is given by supply and demand forces, and is set regardless of the manufacturing costs of a portfolio of netting sets.

However, FVA must be used when we want to calculate how much a derivative is worth to us (the *Value to Me*). Without it, a business unit will not be able to make sound business decisions. Not using FVA for this would be like an oil company deciding on an extraction project only from the market price of the oil inside the oil bag, without considering the extraction costs.

Because of this, CVA and FVA should not introduce any double-counting if done properly. Each serves its purpose. Thinking that there is double-counting is like thinking that estimating the *Value to Me* of a construction project for luxury flats (ie. substracting its manufacturing cost from its sale market price) contains double-counting because the market price already accounts for the fact that they are 'luxury' flats.

We have seen how rehypothication, netting and the overall hedging strategy that an institution has are going to have different impact in CVA and FVA.

With all this in mind, a proposed framework for counterparty and funding risk management in a financial institution is shown in Figure 5. The organisation could set up two specialised desks: the CVA and the FVA desk. Each of them take on the default and funding risk the dealing desks create, so they (the dealing desks) can forget about default and funding matters and concentrate in what they know best: generating risk-neutral derivatives, selling them, and hedging their market risk.

The CVA desk will charge a default insurance fee at trade inception to the dealing desks, which should be the incremental CVA that each trades brings to the bank. Then, it will manage default risk and CVA volatility in the balance sheet, an any profit and loss coming from it will be attributed to the CVA desk[14].

Similariy, the FVA desk will charge a funding insurance fee at trade inception to the dealing desks, which should be the incremental FVA that each trades brings to the bank. Then, it will also manage all funding risks and it will be in charge of all the funding needs that the dealing desks might have in the future[15]. In the case of the FVA desk, the 'market' that it uses to source the needed funding could be external investors, or an internal treasury desk. The FVA calculation and management strategy should reflect which set up it operates under.

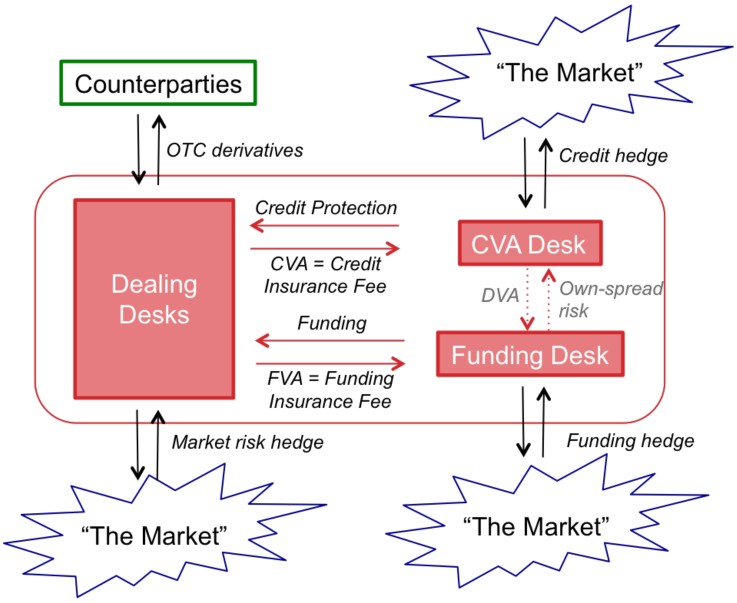

Figure 5: Illustration of a proposed counterparty and funding risk management framework in a large financial institution.

Each CVA and FVA desk will have the option of hedging their respective risks, or not. The CVA desk will be theoretically able to hedge its risk mainly by setting up long-term default protection positions against the market. Similarly, the FVA desk will be able to hedge its risk mainly by setting up long-term funding protection positions. However, these desks may decide also not to hedge their risk, or to hedge them only partially, given the market restrictions. This last option is the most common. There is no right

[14]As far as hedging is possible given the market's restrictions.
[15]Also, as far as hedging is possible given the market's restrictions.

or wrong solution solution as to which hedging strategy is best, as long as we each desk knows what they are doing. The only factor that makes a decision truly wrong in this matter is not knowing the implications of a decision.

Further to it, depending on how bilateral CVA is managed by the organisation, and given that changes in the liability side of CVA and in FVA are highly related, the funding desk may be able to help hedging some part of the DVA_{liab} component of the volatility in the balance sheet, via internal CVA-FVA desks transactions.

In this way, each desk attends to what they know best: the dealing desks creates risk-neutral derivatives; the CVA desk manages and hedges credit risk, and the FVA desk does the same with funding risk.

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References

[1] C. Burgard and M. Kjaer, *Generalised cva with funding and collateral via semi-replication.* Dec 2012.

[2] M. Choudhry, *Two curves: Libor, ois and multi-currency option.* www.numerix.com, 2013.

[3] D. Duffie and H. Zhu, *Does a central clearing counterparty reduce counterparty risk?* Mar 2010.

[4] M. Fujii, Y. Shimadaz, and A. Takahashi, *A note on construction of multiple swap curves with and without collateral.* www.ssrn.com, 2009.

[5] J. Hull and A. White, *Collateral and credit issues in derivatives pricing.* Jan 2013.

[6] A. Pallavicini, D. Perini, and D. Brigo, *Funding value adjustment: a consistent framework including cva, dva, collateral, netting rules and re-hypothication.* Dec 2012.

[7] V. Peterbarg, *Funding beyond discounting: collateral agreements and derivatives pricing*, Risk, (2010).

[8] I. Ruiz, *Fva demystified - cva, dva, fva and their interaction (part i).* www.iruizconsulting.com, 2013.

[9] I. Ruiz, R. Pachón, and P. del Boca, *Optimal right and wrong way risk.* www.iruizconsulting.com, April 2013.